RETRIEVAL OF ATMOSPHERIC STATIC STABILITY FROM MST RADAR RETURN SIGNAL POWER

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1. Introduction

The vertical gradient of potential temperature gives a measure of the atmosphere's static stability, i.e. of its resistance to vertical motions. It is conveniently quantified in terms of the square of the Brunt-Väisälä frequency, ω_B^2 (rad^2 s^{-2}):

$$\omega_B^2 = \frac{g}{\theta}\frac{\partial\theta}{\partial z} = g\left(\frac{\partial \ln\theta}{\partial z}\right) \tag{1}$$

where θ (K) is potential temperature [$= T(1000/p)^{2/7}$], T (K) is absolute temperature, p (hPa) is pressure, g (m s^{-2}) is gravitational acceleration and z (m) is altitude. Perturbations of ω_B^2 values are commonly seen in association with a variety of atmospheric phenomena including fronts, gravity waves and turbulence. Moreover, the variations of ω_B^2 as a function of altitude have an effect on both gravity wave propagation and turbulence generation. Routinely derived profiles of this parameter will therefore have a number of uses.

The purpose of this paper is to describe a method of retrieving profiles of ω_B^2 based on the MST radar return signal power, P, for a vertically directed beam. This method has much in common with that described by *Gage and Green* (1982). Data are considered from observations made by the ESRAD MST radar (*Chilson et al.*, 1999), which is located at Esrange, the Swedish Space Corporation's rocket range, in northern Sweden (67.9°N, 21.1°E). Comparisons are made between values of ω_B^2 derived from the retrieval method and those derived from measurements made by 221 radiosondes launched from the same site during the winters of 1996/1997 to 1999/200. In deriving the radiosonde values of ω_B^2 using Equation 1, gradients of potential temperature are evaluated over a vertical interval of 300 m and subsequently transferred to a vertical grid, between 1.0 and 15.7 km altitude at 300 m intervals, so as to correspond to the radar measurements.

2. Method

It has long been appreciated that perturbations of P and ω_B^2 are closely related. The theory of Fresnel scatter (*Gage et al.*, 1981) gives a direct relationship between the two through consideration of the mean vertical gradient of generalised potential refractive index, M (*Ottersten*, 1969). The latter is often conveniently approximated by the dry term, M_D:

$$M_D = -77.6 \times 10^{-6}\frac{p}{T}\frac{\partial \ln\theta}{\partial z} \tag{2}$$

although the full term additionally depends on the both the specific humidity, q (g kg^{-1}), and its vertical gradient:

$$M = M_D \left[1 + \frac{15500q}{T} - \frac{\frac{7800}{T}\frac{\partial q}{\partial z}}{\frac{\partial \ln \theta}{\partial z}} \right] \tag{3}$$

It can be seen that the dry approximation is valid when the second and third terms in the square brackets of Equation 3 are significantly less than 1; this is typically assumed to be the case above the first few kilometres of the atmosphere. This assumption will be examined in greater detail shortly. For the time being the humidity contributions will be ignored entirely.

It will be recognised that the p/T term in Equation 2 is proportional to density, which can be approximated as $\rho_0 \exp(-z/\overline{H})$ where ρ_0 (hPa) is the pressure at mean sea level and $\overline{H}$ (m) is the mean scale height across the considered altitude range. For the data considered in present investigation, the value of H, which is given by RT/g, where R is the gas constant for dry air, is typically around 8 km at an altitude of 1 km and decreases to around 6 km at the tropopause level; it remains approximately constant at this value between the tropopause and an altitude of 15.7 km. A mean value of 6.71 km is calculated over all data points. Assuming ρ_0 = 1000 hPa, the observed and assumed values of density are typically within 10% of each other at all altitudes within the range 1.0 - 15.7 km.

Combining the expectation that $P \propto M^2/z^2$ (*Gage et al.*, 1981) with Equation 2, and substituting for Equation 1, gives:

$$P \propto \frac{\left[\exp(-z/\overline{H})\omega_B^2\right]^2}{z^2} \tag{4}$$

which can be rearranged in order to define a radar factor r_B^2:

$$r_B^2 = z_{km} \exp(z_{km}/\overline{H}_{km})\sqrt{P} \tag{5}$$

where the altitude above mean sea level, z_{km}, and the mean scale height, $\overline{H}_{km}$, are both given in units of kilometres, such that the following linear relationship is expected:

$$\omega_B^2 = g_0 r_B^2 + y_0 \tag{6}$$

3. Results

Figure 1(left panel) shows the relationship between radar-derived values of r_B^2 and radiosonde-derived values of ω_B^2. The plot area is divided into a 100 by 100 grid, with divisions of 0.1×10^{-4} rad^2 s^{-2} along the y-axis and 0.1×10^3 arbitrary r_B^2 units along the x-axis; the grey scale represents the number of data points falling within each cell. There is clearly a high degree of correlation between the two sets of values and two distinct clusters of data points can be seen; those with small values of ω_B^2 and of r_B^2, which correspond to tropospheric measurements, and those with larger values of both, which correspond to lower stratospheric measurements. There is, nevertheless, considerable scatter around these clusters.

Much of this scatter, for the tropospheric measurements, can be accounted for by the fact that the humidity contributions to M have been ignored in the retrieval model. The values of r_B^2 are, in fact, overestimated by a factor $|M/M_D|$, i.e. by the modulus of the square bracket in Equation 3. The validity of the retrieval algorithm is therefore limited to those altitudes at which the humidity contributions to M can ignored. Since the necessary information is available from the radiosonde measurements, humidity-corrected values of radar-derived r_B^2,

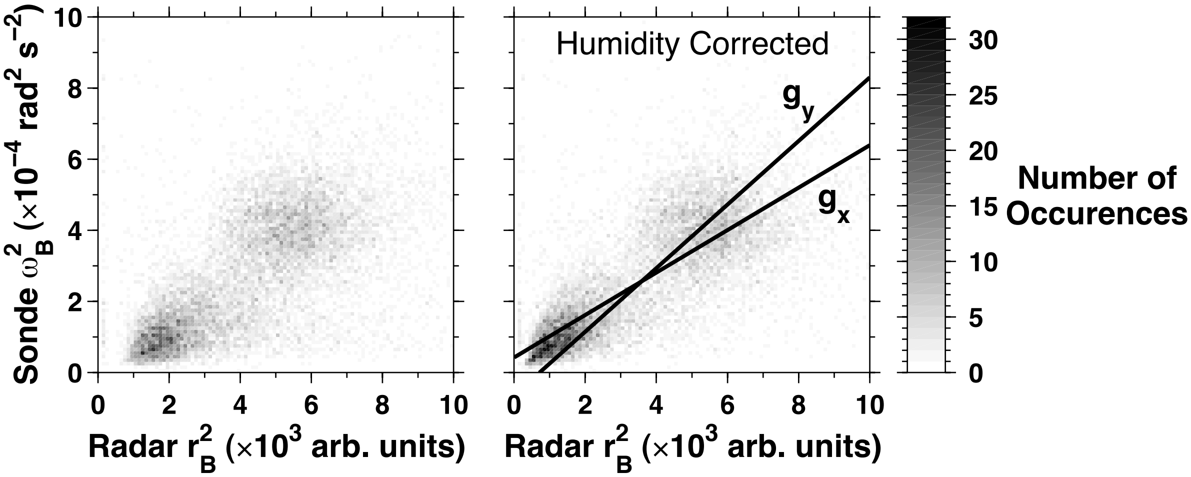

Figure 1: The relationship between radiosonde-derived values of ω_B^2 and radar-derived values of r_B^2 (left panel). The values of r_B^2 shown in the right hand panel have been humidity corrected.

i.e. $|M_D/M| \times r_B^2$, are shown in Figure 1(right panel). Clearly the oversimplification accounts many of the largest deviations from a linear relationship associated with small values of radiosonde-derived ω_B^2.

The best fit between radiosonde-derived values of ω_B^2 and humidity corrected radar-derived values of r_B^2 is found following the method of *Hocking et al.* (2001). The lines with gradients g_x and g_y shown in Figure 1(right panel) represent, respectively, the least squares best fits from the regression of y on x and *vice versa*. These correspond, respectively, to the assumptions that all of the variability is associated with the radiosonde-derived values of ω_B^2, and that there are no errors associated with the humidity corrected radar-derived values of r_B^2, and *vice versa*. In addition to the fact that there are measurement errors associated with both the radar and the radiosondes, each instrument is measuring in different regions of the atmosphere.

Despite the fact that the radiosondes are launched from the radar site, they can drift by up to 100 km downwind by the time that they reach an altitude of 15.7 km. In one extreme case (not shown) the profiles of radiosonde-derived values of ω_B^2 in the lower-stratosphere show large perturbations as a function of altitude whereas the profiles of radar-derived r_B^2 do not. The perturbations of radiosonde-derived ω_B^2 are attributed to mountain wave activity since they are anticorrelated with large perturbations of the balloon ascent rate. The poor correlation between the radar and radiosonde profiles is attributed to the spatially localised nature of mountain wave activity; the radiosonde was 50 km from the radar site by the time that it reached the tropopause level.

Although neither of the lines shown in Figure 1(right panel) represents the best fit between the data, each one represents an extreme of the possible fits. The required best fit lies somewhere between the two. It is speculated that the physical separation between the radar and radiosonde measurements is responsible for a large amount of the scatter seen in Figure 1(right panel). The value of g_0 is therefore selected such that the variability associated with each parameter is equal, i.e. to the condition $\sigma_y = g_0\sigma_x$ (= 7.12×10^{-5} rad^2 s^{-2}) following the nomenclature of *Hocking et al.* (2001).

Figure 2 compares profiles of ω_B^2 derived from radiosonde data (thick grey lines) and radar

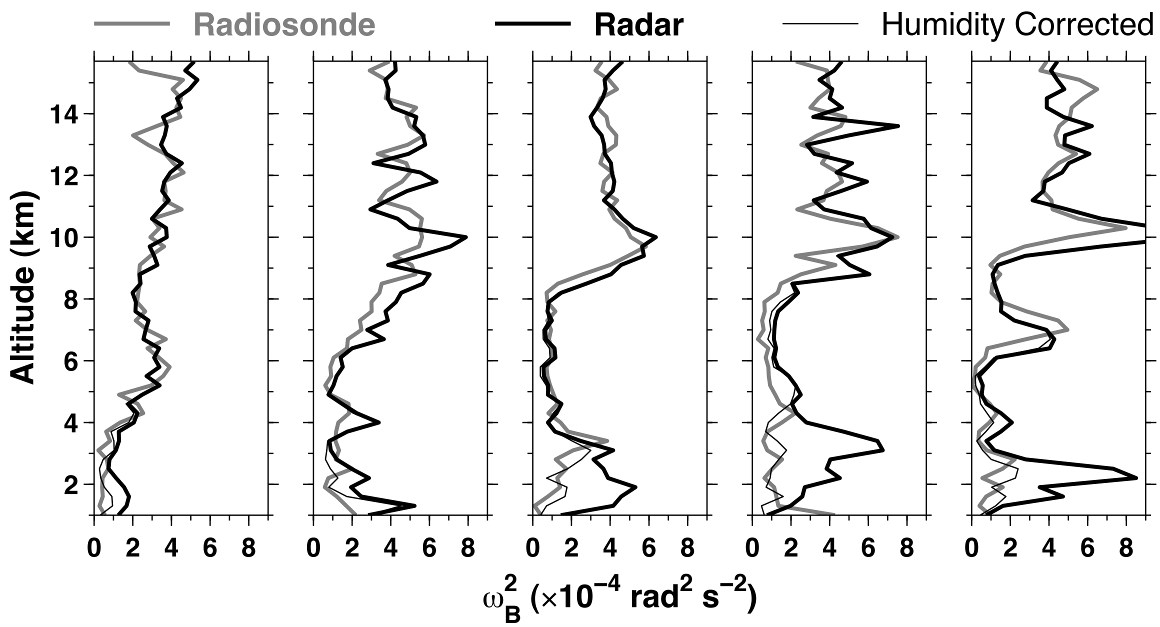

Figure 2: A comparison of profiles of ω_B^2 derived from radiosonde data, radar data and humidity corrected radar data.

data (thick black lines) for 5 individual cases. They are clearly quantitatively and qualitatively well matched. The only significant discrepancies occur in the lower troposphere and, as can be seen from the corresponding profiles of humidity corrected radar-derived values (thin black lines), these can be attributed to the over-simplification of assuming $M = M_D$ inherent in the retrieval model. The question remains as to lowest altitude above which the assumption that $M = M_D$ is always valid. The contributions of the humidity terms to M, for specific profiles, can be significant right the way up to the tropopause level. In general, therefore, the retrieval method can only be assumed to be valid at lower-stratospheric altitudes. However, under specific circumstances, the retrieval method can be used at some, or all, altitudes below the tropopause.

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